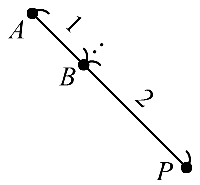
113 學年度四技二專第一次聯合模擬考試
共同科目　數學(C)卷　詳解

數學(C)卷　　113-1-C

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
D	B	A	C	A	D	C	B	A	D	C	D	A	B	B	C	D	A	C	B	C	A	C	B	D

1. $\because \overline{AP}:\overline{PB}=3:2$ 且 P 不在 A 、 B 之間

$\therefore$三點之位置為 $A-B-P$ 且 $\overline{AB}:\overline{BP}=1:2$



令 $P(x,y)$

由分點公式知 $(4,-7)=(\dfrac{1\cdot x+2\times(-6)}{3},\dfrac{1\cdot y+2\times 3}{3})$

$\therefore \begin{cases}\dfrac{x-12}{3}=4\\ \dfrac{6+y}{3}=-7\end{cases} \Rightarrow \begin{cases}x=24\\ y=-27\end{cases}$　$\therefore P(24,-27)$，故選(D)

2. 小英與 y 軸之距離 $=4\times 2=8$

$\because$小英在第二象限　$\therefore A$ 點坐標為 $(-8,4)$，故選(B)

3. $\sin 1590^\circ=\sin(360^\circ\times 4+150^\circ)=\sin 150^\circ=\sin 30^\circ=\dfrac{1}{2}$

$\cos(-1860^\circ)=\cos(-360^\circ\times 6+300^\circ)=\cos 300^\circ$

$=\cos 60^\circ=\dfrac{1}{2}$

$\tan 1395^\circ=\tan(360^\circ\times 3+315^\circ)=\tan 315^\circ$

$=-\tan 45^\circ=-1$

$\tan(-960^\circ)=\tan(-360^\circ\times 3+120^\circ)=\tan 120^\circ$

$=-\tan 60^\circ=-\sqrt{3}$

$\therefore$原式 $=\dfrac{1}{2}\times\dfrac{1}{2}+(-1)\times(-\sqrt{3})=\dfrac{1}{4}+\sqrt{3}$，故選(A)

4. 最大負同界角 $\alpha=-\dfrac{25\pi}{6}+2\pi\times 2=-\dfrac{\pi}{6}$

最小正同界角 $\beta=-\dfrac{25\pi}{6}+2\pi\times 3=\dfrac{11\pi}{6}$

$\therefore 5\alpha+\beta=5\times(-\dfrac{\pi}{6})+\dfrac{11\pi}{6}=\pi$，故選(C)

5. 原式為 $2\vec{a}-3(\vec{b}-2\vec{x})=5\vec{x}+\vec{a}-\vec{b}$

$\Rightarrow 2\vec{a}-3\vec{b}+6\vec{x}=5\vec{x}+\vec{a}-\vec{b}$

$\Rightarrow \vec{x}=-\vec{a}+2\vec{b}=-(1,-4)+2(3,-5)=(5,-6)$

$\therefore \alpha+\beta=5+(-6)=-1$，故選(A)

6. $\overrightarrow{AD}=(x-1,y+2)$，$\overrightarrow{AB}=(2,-1)$

$\because \overrightarrow{AD}\perp\overrightarrow{AB}$　$\therefore \overrightarrow{AD}\cdot\overrightarrow{AB}=0$

$\Rightarrow 2(x-1)-(y+2)=0\Rightarrow 2x-y=4\cdots$①

又 $\overrightarrow{BD}=(x-3,y+3)$，$\overrightarrow{AC}=(1,3)$

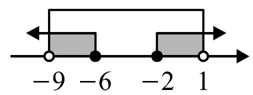
$\because \overrightarrow{BD}\,//\,\overrightarrow{AC}$　$\therefore \dfrac{x-3}{1}=\dfrac{y+3}{3}\Rightarrow 3x-y=12\cdots$②

由①、②知：$x=8$，$y=12$　$\therefore x+y=20$，故選(D)

7. $2\le|x+4|<5$

$\Rightarrow \begin{cases}|x+4|\ge 2\\ |x+4|<5\end{cases} \Rightarrow \begin{cases}x+4\ge 2 \text{ 或 } x+4\le -2\\ -5<x+4<5\end{cases}$

$\Rightarrow \begin{cases}x\ge -2 \text{ 或 } x\le -6\\ -9<x<1\end{cases}$



$\therefore -9<x\le -6$ 或 $-2\le x<1\Rightarrow x=-8$、$-7$、$-6$、$-2$、$-1$、$0$ 共有 6 個整數解，故選(C)

8. $\because -\dfrac{1}{2}<x<3$　$\therefore (2x+1)(x-3)<0\Rightarrow 2x^2-5x-3<0$

與 $2x^2+ax+b<0$ 比較係數後得知：$a=-5$ 且 $b=-3$

$\therefore a-2b=-5-2\times(-3)=1$，故選(B)

9. 由算幾不等式知：$\dfrac{3a+4b}{2}\ge\sqrt{3a\cdot 4b}\Rightarrow\dfrac{10}{2}\ge\sqrt{12ab}$

$\Rightarrow 25\ge 12ab\Rightarrow ab\le\dfrac{25}{12}$　$\therefore$最大值 $=\dfrac{25}{12}$，故選(A)

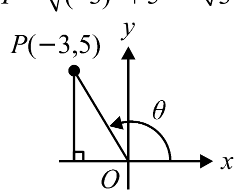
10. 1 弧度 $=\dfrac{180^\circ}{\pi}\fallingdotseq 57^\circ$

$\therefore a=\cos 1\fallingdotseq\cos 57^\circ=\sin 33^\circ$，$b=\sin 1\fallingdotseq\sin 57^\circ$

$c=\tan 1\fallingdotseq\tan 57^\circ>1$，$d=\cos\dfrac{1}{2}\fallingdotseq\cos 28.5^\circ=\sin 61.5^\circ$

可知：$c>d>b>a$，故選(D)

11. $\overline{OP}=\sqrt{(-3)^2+5^2}=\sqrt{34}$



(A) $\sin\theta=\dfrac{5}{\sqrt{34}}$

(B) $\cos\theta=\dfrac{-3}{\sqrt{34}}$

(C) $\cos(90^\circ+\theta)=-\sin\theta=-\dfrac{5}{\sqrt{34}}<0$

(D) $\tan(180^\circ-\theta)=-\tan\theta=\dfrac{5}{3}>0$

故選(C)

12. (A) $\because$振幅為 2　$\therefore a=2$

(B) $\because$週期$=\pi$　$\therefore \dfrac{2\pi}{|b|}=\pi \Rightarrow |b|=2 \Rightarrow b=\pm 2$

又$b>0$　$\therefore b=2$

(C) 平衡位置爲 $y=2$　$\therefore c=2$

(D) $\because$ $y=3$ 之直線與圖形有 4 個交點

$\therefore$方程式 $f(x)=3$ 有 4 個相異實數解，故選(D)

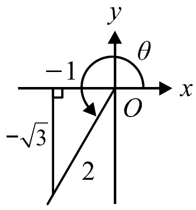
13. $4\sin^2\theta+8\cos\theta+1=0 \Rightarrow 4(1-\cos^2\theta)+8\cos\theta+1=0$

$\Rightarrow 4\cos^2\theta-8\cos\theta-5=0$

$\Rightarrow (2\cos\theta+1)(2\cos\theta-5)=0$

$\therefore \cos\theta=\dfrac{-1}{2}\ or\ \dfrac{5}{2}$(不合)

又$\because \cos\theta<0$ 且 $\tan\theta>0$　$\therefore \theta$是第 3 象限角



$\therefore \cos(270^\circ+\theta)=\sin\theta=\dfrac{-\sqrt{3}}{2}$，故選(A)

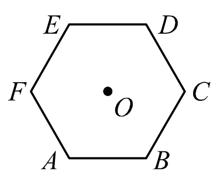
14. $\overrightarrow{PQ}=\overrightarrow{PB}+\overrightarrow{BQ}=\dfrac{2}{5}\overrightarrow{CB}+(-\dfrac{1}{2})\overrightarrow{AB}$

$=\dfrac{2}{5}(\overrightarrow{CA}+\overrightarrow{AB})-\dfrac{1}{2}\overrightarrow{AB}=\dfrac{2}{5}\overrightarrow{AB}-\dfrac{1}{2}\overrightarrow{AB}+\dfrac{2}{5}\overrightarrow{CA}$

$=\dfrac{-1}{10}\overrightarrow{AB}-\dfrac{2}{5}\overrightarrow{AC}$　$\therefore x=\dfrac{-1}{10}$ 且 $y=\dfrac{-2}{5}$

$\therefore 10x+5y=10\times(\dfrac{-1}{10})+5\times(\dfrac{-2}{5})=-3$，故選(B)

15. 甲：$\overrightarrow{AB}+\overrightarrow{BC}=\overrightarrow{AC}$



乙：$\overrightarrow{AB}+\overrightarrow{AF}=\overrightarrow{AO}$

丙：$\overrightarrow{AB}-\overrightarrow{AE}=\overrightarrow{AB}+\overrightarrow{EA}$

$=\overrightarrow{EA}+\overrightarrow{AB}=\overrightarrow{EB}$

丁：$\overrightarrow{AC}-\overrightarrow{AE}=\overrightarrow{AC}+\overrightarrow{EA}=\overrightarrow{EA}+\overrightarrow{AC}=\overrightarrow{EC}$

$\because |\overrightarrow{EB}|>|\overrightarrow{EC}|=|\overrightarrow{AC}|>|\overrightarrow{AO}|$

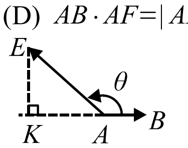
$\therefore$乙同學的合成向量最短，故選(B)

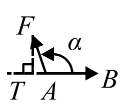
16. (A) $\overrightarrow{AB}\cdot\overrightarrow{AC}>0$

(B) $\overrightarrow{AB}\cdot\overrightarrow{AD}>0$

(C) $\overrightarrow{AB}\cdot\overrightarrow{AE}=|\overrightarrow{AB}||\overrightarrow{AE}|\cos\theta=-|\overrightarrow{AB}||\overrightarrow{AK}|$

(D) $\overrightarrow{AB}\cdot\overrightarrow{AF}=|\overrightarrow{AB}||\overrightarrow{AF}|\cos\alpha=-|\overrightarrow{AB}||\overrightarrow{AT}|$





$\because |\overrightarrow{AK}|>|\overrightarrow{AT}|$　$\therefore \overrightarrow{AB}\cdot\overrightarrow{AE}$ 爲最小，故選(C)

17. $0.3\overline{5}=\dfrac{35-3}{90}=\dfrac{32}{90}=\dfrac{16}{45}$

$\therefore a\times\dfrac{16}{45}-\dfrac{1}{6}=a\times 0.35 \Rightarrow a\times\dfrac{16}{45}-\dfrac{1}{6}=\dfrac{7}{20}a$

$\times 180$

$\Rightarrow\ 64a-30=63a \Rightarrow a=30$，故選(D)

[另解]

$a\times 0.3\overline{5}-a\times 0.35=\dfrac{1}{6} \Rightarrow a\times 0.35\overline{5}-a\times 0.35=\dfrac{1}{6}$

$\Rightarrow a\times 0.00\overline{5}=\dfrac{1}{6} \Rightarrow a\times\dfrac{5}{900}=\dfrac{1}{6} \Rightarrow a=30$，故選(D)

18. (A) $\because$拋物線開口向上　$\therefore a>0$

$\because$對稱軸 $x=\dfrac{b}{2a}<0$　$\therefore b<0$

(B) $\because$拋物線與 y 軸交於負向　$\therefore c<0$

(C) $\because$拋物線與 x 軸交兩點

$\therefore (-b)^2-4ac>0 \Rightarrow b^2-4ac>0$

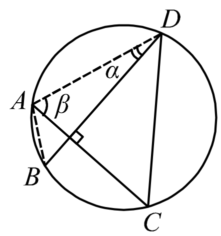
(D) $\because$拋物線上一點 $(1, a-b+c)$ 在 x 軸上方

$\therefore a-b+c>0$

故選(A)

19. 連接 $\overline{AB}$ 、 $\overline{AD}$

令$\angle ADB=\alpha$ 、 $\angle DAC=\beta$，如下圖



在ΔACD 中，由正弦定理知：

$\dfrac{\overline{CD}}{\sin\beta}=2\times 4 \Rightarrow \sin\beta=\dfrac{3}{4}$

在ΔABD 中，由正弦定理知：

$\dfrac{\overline{AB}}{\sin\alpha}=2\times 4 \Rightarrow \overline{AB}=8\sin\alpha$

又$\because \alpha+\beta=90^\circ$

$\therefore \sin\alpha=\sin(90^\circ-\beta)=\cos\beta=\dfrac{\sqrt{7}}{4}$

$\therefore \overline{AB}=8\times\dfrac{\sqrt{7}}{4}=2\sqrt{7}$，故選(C)

20. ΔABD 中　$\because \cos(\angle ADB)=\dfrac{2^2+3^2-\sqrt{7}^2}{2\times 2\times 3}=\dfrac{1}{2}$

$\therefore \cos(\angle ADC)=\cos(180^\circ-\angle ADB)=-\cos(\angle ADB)=-\dfrac{1}{2}$

令$\overline{CD}=x$

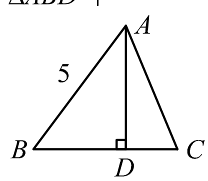
$\therefore \Delta ACD$ 中 $\cos(\angle ADC)=\dfrac{3^2+x^2-7^2}{2\times 3\times x}=-\dfrac{1}{2}$

$\Rightarrow 2x^2-80=-6x \Rightarrow x^2+3x-40=0$

$\Rightarrow (x+8)(x-5)=0$

$\therefore x=5$ or -8(不合)，故選(B)

21. ΔABD 中



$\because \overline{AB}=5$ ，$\sin B=\dfrac{4}{5}$

$\therefore \overline{AD}=\overline{AB}\times\sin B=5\times\frac{4}{5}=4\Rightarrow\overline{BD}=\sqrt{5^2-4^2}=3$

在 ΔACD 中

$\because \overline{AD}=4$，$\sin C=\frac{12}{13}=\frac{\overline{AD}}{\overline{AC}}$　$\therefore \overline{AC}=\overline{AD}\times\frac{13}{12}=\frac{13}{3}$

$\Rightarrow\overline{CD}=\overline{AC}\times\cos C=\frac{13}{3}\times\frac{5}{13}=\frac{5}{3}$

由上可知：ΔABC面積$=\frac{1}{2}\overline{BC}\times\overline{AD}=\frac{1}{2}(3+\frac{5}{3})\times 4$

$=\frac{28}{3}$，故選(C)

22. $\because \vec{a}$ 與 $\vec{b}$ 互相垂直　$\therefore \vec{a}\cdot\vec{b}=0$

又 $\because (2\vec{a}-\vec{b})\perp(\vec{a}+t\vec{b})$　$\therefore (2\vec{a}-\vec{b})\cdot(\vec{a}+t\vec{b})=0$

$\Rightarrow 2|\vec{a}|^2+(2t-1)\vec{a}\cdot\vec{b}-t|\vec{b}|^2=0\Rightarrow 2|\vec{a}|^2-t|\vec{b}|^2=0$

$\Rightarrow 2\times1^2-t\times2^2=0\Rightarrow t=\frac{1}{2}$，故選(A)

23. $\because \overrightarrow{OD}$在$\overrightarrow{OC}$上之正射影$=\overrightarrow{OA}$在$\overrightarrow{OC}$上之正射影$=\overrightarrow{OB}$

$\therefore \frac{\overrightarrow{OA}\cdot\overrightarrow{OC}}{|\overrightarrow{OC}|^2}\overrightarrow{OC}=(a,b)\Rightarrow\frac{-1\times3+3\times6}{45}(3,6)=(a,b)$

$\Rightarrow a=1$ 且 $b=2$　$\therefore a+b=1+2=3$，故選(C)

24. $\vec{a}\cdot\vec{b}=(x,2)\cdot(1,y)=x+2y$

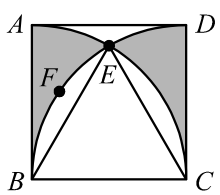
由柯西不等式知：

$[(x-1)^2+(y-2)^2]\cdot(1^2+2^2)\geq[(x-1)+2(y-2)]^2$

$\Rightarrow 5\times5\geq(x+2y-5)^2\Rightarrow -5\leq x+2y-5\leq 5$

$\Rightarrow 0\leq x+2y\leq 10$　$\therefore x+2y$ 之最小值爲 0，故選(B)

25.



$\because \overline{BE}=\overline{EC}=\overline{BC}=1$

$\therefore \Delta BEC$ 爲正三角形 $\angle EBC=60^\circ$ 可知 $\angle ABE=30^\circ$

扇形 ABE 之面積$=1\times1\times\pi\times\frac{30}{360}=\frac{\pi}{12}$

弓形 BEF 之面積$=$扇形$CEB-\Delta BCE$

$=1\times1\times\pi\times\frac{60}{360}-\frac{\sqrt{3}}{4}\times1^2=\frac{\pi}{6}-\frac{\sqrt{3}}{4}$

$\therefore$所求之塗色面積$=2\times[\frac{\pi}{12}-(\frac{\pi}{6}-\frac{\sqrt{3}}{4})]=\frac{\sqrt{3}}{2}-\frac{\pi}{6}$

故選(D)